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Constant-Cutoff Approach to Radiative Decays of Hyperons: E2/M1 Transition Ratios

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We suggest a quantum stabilization method for the $SU(2)$ σ-model, based on the constant-cutoff limit of the cutoff quantization method developed by Balakrishna *et al.*, which avoids the difficulties with the usual soliton boundary conditions pointed out by Iwasaki and Ohyama. We investigate the baryon number $B = 1$ sector of the model and show that after the collective coordinate quantization it admits a stable soliton solution which depends on a single dimensional arbitrary constant. We then study the radiative decays of $J^{\pi} = \frac{3}{2}^{+}$ baryons using the constant-cutoff approach to the $SU(3)$ collective treatment of the Skyrme model for hyperons. Thus we evaluate the widths and E2/M1 ratios, showing that there is a general qualitative agreement with the results obtained using the complete Skyrme model, as well as the nonrelativistic quark model and quenched lattice model, for the total widths.

1. INTRODUCTION

It was shown by Skyrme (1961, 1962) that baryons can be treated as solitons of a nonlinear chiral theory. The original Lagrangian of the chiral $SU(2)$ σ-model is

$$\mathcal{L} = \frac{F_{\pi}^2}{16} \operatorname{Tr} \partial_{\mu} U \, \partial^{\mu} U^{+} \tag{1.1}$$

where

$$U = \frac{2}{F_{\pi}} (\sigma + i\boldsymbol{\tau} \cdot \boldsymbol{\pi}) \tag{1.2}$$

is a unitary operator ($UU^{+} = 1$) and F_{π} is the pion-decay constant. In (1.2) $\sigma = \sigma(\mathbf{r})$ is a scalar meson field and $\boldsymbol{\pi} = \boldsymbol{\pi}(\mathbf{r})$ is the pion isotriplet.

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The classical stability of the soliton solution to the chiral σ-model Lagrangian requires an additional ad hoc term, proposed by Skyrme (1961, 1962), to be added to (1.1):

$$\mathcal{L}_{\mathrm{Sk}} = \frac{1}{32e^2}\,\mathrm{Tr}[U^{+}\partial_{\mu}U,\, U^{+}\partial_{\nu}U]^2 \tag{1.3}$$

with a dimensionless parameter e and where $[A, B] = AB - BA$. It was shown by several authors (Adkins *et al.*, 1983; Witten, 1979, 1983a,b) that, after the collective quantization using the spherically symmetric ansatz

$$U_0(\mathbf{r}) = \exp[i\boldsymbol{\tau}\cdot\mathbf{r}_0 F(r)], \qquad \mathbf{r}_0 = \mathbf{r}/r \tag{1.4}$$

the chiral model, with both (1.1) and (1.3) included, gives good agreement with experiment for several important physical quantities. Thus it should be possible to derive the effective chiral Lagrangian, obtained as a sum of (1.1) and (1.3), from a more fundamental theory like QCD. On the other hand, it is not easy to generate a term like (1.3) and give a clear physical meaning to the dimensionless constant e in (1.3) using QCD.

Mignaco and Wulck (1989) (MW) indicated therefore a possibility to build a stable single baryon ($n = 1$) quantum state in the simple chiral theory with the Skyrme stabilizing term (1.3) omitted. They showed that the chiral angle $F(r)$ is in fact a function of a dimensionless variable $s = \frac{1}{2}\chi''(0)r$, where $\chi''(0)$ is an arbitrary dimensional parameter intimately connected to the usual stability argument against the soliton solution for the nonlinear σ-model Lagrangian.

Using the adiabatically rotated ansatz $U(\mathbf{r}, t) = A(t)U_0(\mathbf{r})A^{+}(t)$, where $U_0(\mathbf{r})$ is given by (1.4), MW obtained the total energy of the nonlinear σ-model soliton in the form

$$E = \frac{\pi}{4}F_{\pi}^2\frac{1}{\chi''(0)}\,a + \frac{1}{2}\frac{[\chi''(0)]^3}{(\pi/4)F_{\pi}^2 b}\,J(J+1) \tag{1.5}$$

where

$$a = \int_0^{\infty}\left[\frac{1}{4}s^2\left(\frac{d\mathcal{F}}{ds}\right)^2 + 8\sin^2\left(\frac{1}{4}\mathcal{F}\right)\right]ds \tag{1.6}$$

$$b = \int_0^{\infty} ds\,\frac{64}{3}\,s^2\sin^2\left(\frac{1}{4}\mathcal{F}\right) \tag{1.7}$$

and $\mathcal{F}(s)$ is defined by

$$F(r) = F(s) = -n\pi + \tfrac{1}{4}\mathcal{F}(s) \tag{1.8}$$

The stable minimum of the function (1.5) with respect to the arbitrary dimensional scale parameter $\chi''(0)$ is

$$E = \frac{4}{3} F_\pi \left[\frac{3}{2} \left(\frac{\pi}{4} \right)^2 \frac{a^3}{b} J(J+1) \right]^{1/4} \tag{1.9}$$

Despite the nonexistence of the stable classical soliton solution to the nonlinear σ-model, it is possible, after the collective coordinate quantization, to build a stable chiral soliton at the quantum level, provided that there is a solution $F = F(r)$ which satisfies the soliton boundary conditions, i.e., $F(0) = -n\pi$, $F(\infty) = 0$, such that the integrals (1.6) and (1.7) exist.

However, as pointed out by Iwasaki and Ohyama (1989), the quantum stabilization method in the form proposed by Mignaco and Wulck (1989) is not correct, since in the simple σ-model the conditions $F(0) = -n\pi$ and $F(\infty) = 0$ cannot be satisfied simultaneously. In other words, if the condition $F(0) = -\pi$ is satisfied, Iwasaki and Ohyama obtained numerically $F(\infty) \to -\pi/2$, and the chiral phase $F = F(r)$ with correct boundary conditions does not exist.

Iwasaki and Ohyama also proved analytically that both boundary conditions $F(0) = -n\pi$ and $F(\infty) = 0$ cannot be satisfied simultaneously. Introducing a new variable $y = 1/r$ into the differential equation for the chiral angle $F = F(r)$, we obtain

$$\frac{d^2F}{dy^2} = \frac{1}{y^2} \sin 2F \tag{1.10}$$

There are two kinds of asymptotic solutions to equation (1.10) around the point $y = 0$, which is called a regular singular point if $\sin 2F \approx 2F$. These solutions are

$$F(y) = \frac{m\pi}{2} + cy^2, \qquad m = \text{even integer} \tag{1.11}$$

$$F(y) = \frac{m\pi}{2} + \sqrt{cy} \cos\left[\frac{\sqrt{7}}{2} \ln(cy) + \alpha \right], \qquad m = \text{odd integer} \tag{1.12}$$

where c is an arbitrary constant and α is a constant to be chosen appropriately. When $F(0) = -n\pi$, then we want to know which of these two solutions are approached by $F(y)$ when $y \to 0$ ($r \to \infty$). In order to answer that question, we multiply (1.10) by $y^2F'(y)$, integrate with respect to y from y to ∞, and use $F(0) = -n\pi$. Thus we get

$$y^2F'(y) + \int_y^\infty 2y[F'(y)]^2 \, dy = 1 - \cos[2F(y)] \tag{1.13}$$

Since the left-hand side of (1.13) is always positive, the value of $F(y)$ is always limited to the interval $n\pi - \pi < F(y) < n\pi + \pi$. Taking the limit $y \to 0$, we find that (1.13) is reduced to

$$\int_0^\infty 2y[F'(y)]^2\,dy = 1 - (-1)^m \tag{1.14}$$

where we used (1.11)–(1.12). Since the left-hand side of (1.14) is strictly positive, we must choose an odd integer m. Thus the solution satisfying $F(0) = -n\pi$ approaches (1.12) and we have $F(\infty) \neq 0$. The behavior of the solution (1.11) in the asymptotic region $y \to \infty$ $(r \to 0)$ is investigated by multiplying (1.10) by $F'(y)$, integrating from 0 to y, and using (1.11). The result is

$$[F'(y)]^2 = \frac{2\sin^2 F(y)}{y^2} + \int_0^y \frac{2\sin^2 F(y)}{y^3}\,dy \tag{1.15}$$

From (1.15) we see that $F'(y) \to$ const as $y \to \infty$, which means that $F(r) \simeq 1/r$ for $r \to 0$. This solution has a singularity at the origin and cannot satisfy the usual boundary condition $F(0) = -n\pi$.

In Dalarsson (1991a,b, 1992), I suggested a method to resolve this difficulty by introducing a radial modification phase $\varphi = \varphi(r)$ in the ansatz (1.4) as follows:

$$U(\mathbf{r}) = \exp[i\boldsymbol{\tau}\cdot\mathbf{r}_0 F(r) + i\varphi(r)], \qquad \mathbf{r}_0 = \mathbf{r}/r \tag{1.16}$$

Such a method provides a stable chiral quantum soliton, but the resulting model is an entirely noncovariant chiral model, different from the original chiral σ-model.

In the present paper we use the constant-cutoff limit of the cutoff quantization method developed by Balakrishna *et al.* (1991; see also Jain *et al.*, 1989) to construct a stable chiral quantum soliton within the original chiral σ-model. Then we apply this method to study the radiative decays of $J^\pi = \frac{3}{2}^+$ baryons using the constant-cutoff version of the $SU(3)$ collective approach to the Skyrme model. Thus we evaluate the widths and E2/M1 ratios, showing that there is a general qualitative agreement with the results obtained using the complete Skyrme model (CSM) (Abada *et al.*, 1996), the nonrelativistic quark model (NRQM) (Darewych *et al.*, 1983; Leinweber *et al.*, 1985), and the quenched lattice model (QLM) (Leinweber *et al.*, 1985).

The reason the cutoff approach to the problem of the chiral quantum soliton works is connected to the fact that the solution $F = F(r)$ which satisfies the boundary condition $F(\infty) = 0$ is singular at $r = 0$. From the physical point of view the chiral quantum model is not applicable to the region about the origin, since in that region there is a quark-dominated bag of the soliton.

However, as argued in Balakrishna *et al.* (1991), when a cutoff ϵ is introduced, the boundary conditions $F(\epsilon) = -n\pi$ and $F(\infty) = 0$ can be satisfied. Balakrishna *et al.* (1991) discuss an interesting analogy with the damped pendulum, showing clearly that as long as $\epsilon > 0$, there is a chiral phase $F = F(r)$ satisfying the above boundary conditions. The asymptotic forms of such a solution are given by their equation (2.2). From these asymptotic solutions we immediately see that for $\epsilon \to 0$ the chiral phase diverges at the lower limit.

Different applications of the constant-cutoff approach are discussed in Dalarsson (1993, 1995a–d, 1996a,c).

2. CONSTANT-CUTOFF STABILIZATION

Substituting (1.4) into (1.1), we obtain for the static energy of the chiral baryon

$$E_0 = \frac{\pi}{2} F_\pi^2 \int_{\epsilon(t)}^{\infty} dr \left[r^2 \left(\frac{dF}{dr} \right)^2 + 2 \sin^2 F \right] \tag{2.1}$$

In (2.1) we avoid the singularity of the profile function $F = F(r)$ at the origin by introducing the cutoff $\epsilon(t)$ at the lower boundary of the space interval $r \in [0, \infty]$, i.e., by working with the interval $r \in [\epsilon, \infty]$. The cutoff itself is introduced following Balakrishna *et al.* (1991) as a dynamic time-dependent variable.

From (2.1) we obtain the following differential equation for the profile function $F = F(r)$:

$$\frac{d}{dr}\left(r^2 \frac{dF}{dr} \right) = \sin 2F \tag{2.2}$$

with the boundary conditions $F(\epsilon) = -\pi$ and $F(\infty) = 0$, such that the correct soliton number is obtained. The profile function $F = F[r; \epsilon(t)]$ now depends implicitly on time t through $\epsilon(t)$. Thus in the nonlinear σ-model Lagrangian

$$L = \frac{F_\pi^2}{16} \int \mathrm{Tr}(\partial_\mu U \, \partial^\mu U^+) \, d^3\mathbf{r} \tag{2.3}$$

we use the ansätze

$$U(\mathbf{r}, t) = A(t) U_0(\mathbf{r}, t) A^+(t), \qquad U^+(\mathbf{r}, t) = A(t) U_0^+(\mathbf{r}, t) A^+(t) \tag{2.4}$$

where

$$U_0(\mathbf{r}, t) = \exp\{i\boldsymbol{\tau} \cdot \mathbf{r}_0 F[r; \epsilon(t)]\} \tag{2.5}$$

The static part of the Lagrangian (2.3), i.e.,

$$L = \frac{F_\pi^2}{16} \int \mathrm{Tr}(\nabla U \cdot \nabla U^+)\, d^3\mathbf{r} = -E_0 \tag{2.6}$$

is equal to minus the energy E_0 given by (2.1). The kinetic part of the Lagrangian is obtained using (2.4) with (2.5) and it is equal to

$$L = \frac{F_\pi^2}{16} \int \mathrm{Tr}(\partial_0 U\, \partial_0 U^+)\, d^3\mathbf{r} = bx^2\, \mathrm{Tr}[\partial_0 A\, \partial_0 A^+] + c[\dot{x}(t)]^2 \tag{2.7}$$

where

$$b = \frac{2\pi}{3} F_\pi^2 \int_1^\infty \sin^2 F\, y^2\, dy, \qquad c = \frac{2\pi}{9} F_\pi^2 \int_1^\infty y^2 \left(\frac{dF}{dy}\right)^2 y^2\, dy \tag{2.8}$$

with $x(t) = [\epsilon(t)]^{3/2}$ and $y = r/\epsilon$. On the other hand, the static energy functional (2.1) can be rewritten as

$$E_0 = ax^{2/3}, \qquad a = \frac{\pi}{2} F_\pi^2 \int_1^\infty \left[y^2 \left(\frac{dF}{dy}\right)^2 + 2 \sin^2 F \right] dy \tag{2.9}$$

Thus the total Lagrangian of the rotating soliton is given by

$$L = c\dot{x}^2 - ax^{2/3} + 2bx^2 \dot{\alpha}_\nu \dot{\alpha}^\nu \tag{2.10}$$

where $\mathrm{Tr}(\partial_0 A\, \partial_0 A^+) = 2\dot{\alpha}_\nu \dot{\alpha}^\nu$ and α_ν ($\nu = 0, 1, 2, 3$) are the collective coordinates defined as in Bhaduri (1988). In the limit of a time-independent cutoff ($\dot{x} \to 0$) we can write

$$H = \frac{\partial L}{\partial \dot{\alpha}^\nu} \dot{\alpha}^\nu - L = ax^{2/3} + 2bx^2\, \dot{\alpha}_\nu \dot{\alpha}^\nu = ax^{2/3} + \frac{1}{2bx^2} J(J+1) \tag{2.11}$$

where $\langle \mathbf{J}^2 \rangle = J(J+1)$ is the eigenvalue of the square of the soliton angular momentum. A minimum of (2.11) with respect to the parameter x is reached at

$$x = \left[\frac{2}{3} \frac{ab}{J(J+1)}\right]^{-3/8} \Rightarrow \epsilon^{-1} = \left[\frac{2}{3} \frac{ab}{J(J+1)}\right]^{1/4} \tag{2.12}$$

The energy obtained by substituting (2.12) into (2.11) is given by

$$E = \frac{4}{3} \left[\frac{3}{2} \frac{a^3}{b} J(J+1)\right]^{1/4} \tag{2.13}$$

This result is identical to the result obtained by Mignaco and Wulck, which is easily seen if we rescale the integrals a and b in such a way that $a \to$

$(\pi/4)F_\pi^2 a$ and $b \to (\pi/4)F_\pi^2 b$ and introduce $f_\pi = 2^{-3/2}F_\pi$. However, in the present approach, as shown in Balakrishna *et al.* (1991), there is a profile function $F = F(y)$ with proper soliton boundary conditions $F(1) = -\pi$ and $F(\infty) = 0$ and the integrals a, b, and c in (2.9)–(2.10) exist and are shown in Balakrishna *et al.* (1991) to be $a = 0.78\ \text{GeV}^2$, $b = 0.91\ \text{GeV}^2$, and $c = 1.46\ \text{GeV}^2$ for $F_\pi = 186$ MeV.

Using (2.13), we obtain the same prediction for the mass ratio of the lowest states as Mignaco and Wulck (1989), which agrees rather well with the empirical mass ratio for the Δ resonance and the nucleon. Furthermore, using the calculated values for the integrals a and b, we obtain the nucleon mass $M(N) = 1167$ MeV, which is about 25% higher than the empirical value of 939 MeV. However, if we choose the pion decay constant equal to $F_\pi = 150$ MeV, we obtain $a = 0.507\ \text{GeV}^2$ and $b = 0.592\ \text{GeV}^2$, giving exact agreement with the empirical nucleon mass.

Finally, it is of interest to know how large the constant cutoffs are for the above values of the pion-decay constant in order to check if they are in the physically acceptable ballpark. Using (2.12), it is easily shown that for the nucleons ($J = 1/2$) the cutoffs are equal to

$$\epsilon = \begin{cases} 0.22\ \text{fm} & \text{for } F_\pi = 186\ \text{MeV} \\ 0.27\ \text{fm} & \text{for } F_\pi = 150\ \text{MeV} \end{cases} \tag{2.14}$$

From (2.14) we see that the cutoffs are too small to agree with the size of the nucleon (0.72 fm), as we should expect, since the cutoffs rather indicate the size of the quark-dominated bag in the center of the nucleon. Thus we find that the cutoffs are of reasonable physical size. Since the cutoff is proportional to F_π^{-1}, we see that the pion-decay constant must be less than 57 MeV in order to obtain a cutoff which exceeds the size of the nucleon. Such values of pion-decay constant are not relevant to any physical phenomena.

3. THE COLLECTIVE APPROACH TO THE *SU*(3) SKYRME MODEL

3.1. Introduction

As argued in Abada *et al.* (1996), the available data on the electromagnetic decays of hyperons, like the reaction $\Delta \to N\gamma$, are rather limited. The ratio of the electric quadrupole (E2) to the magnetic dipole (M1) amplitude, obtained by the $\pi^{0(+)}$-photoproduction experiment at Miami (Abada *et al.*, 1996) is E2/M1 $= (-2.5 \pm 0.2)\%$. For $J = 3/2$ to $J = 1/2$ transitions, which involve strange baryons, the empirical values for the E2/M1 ratios are not available. Nevertheless, these transitions have been studied within several models and in Schat *et al.* (1995a) an analysis of the hyperon radiative decays was made within the framework of the bound-state approach (Callan and Klebanov, 1985; Callan *et al.*, 1988) to the Skyrme (1961, 1962) model. In that approach hyperons are modeled as kaons bound in the background of the static soliton field. For the particular case of $\Lambda(1405)$, in the bound-state approach to the complete Skyrme model and in the constant-cutoff treatment of the bound-state approach, see Schat *et al.* (1995b) and Dalarsson (1996b). However, in Abada *et al.* (1996) hyperons are alternatively described using the $SU(3)$ collective approach to the complete Skyrme model, where the strange degrees of freedom are incorporated as $SU(3)$ collective excitations of the nonstrange soliton to investigate the transitions $B(J = 3/2) \to \gamma B'(J = 1/2)$. In the present section we apply the constant-cutoff approach to the $SU(3)$ collective approach and compare the results with those obtained in the CSM (Abada *et al.*, 1996), the NRQM (Darewych *et al.*, 1983; Leinweber *et al.*, 1985), and the QLM (Leinweber *et al.*, 1985).

3.2. The Effective Interaction

The Lagrangian density for the $SU(3)$ collective model of hyperons is, with Skyrme stabilizing term omitted, given by (Abada *et al.*, 1996)

$$\begin{aligned}\mathcal{L} = {} & \frac{F_\pi^2}{16}\,\mathrm{Tr}\,\partial_\mu U\,\partial^\mu U^+ \\ & - \mathrm{Tr}(T + xS)\,[\beta'(U\,\partial_\mu U\,\partial^\mu U^+ + \partial_\mu U\,\partial^\mu U^+ U^+) + \delta'(U + U^+ - 2)] \\ & - iL_9(\partial_\mu A_\nu - \partial_\nu A_\mu) \\ & \times \mathrm{Tr}\left[\sqrt{U^+}\left(\lambda_3 + \frac{1}{\sqrt{3}}\lambda_8\right)\sqrt{U}(\sqrt{U^+}\,\partial_\mu U\,\partial^\mu U^+\sqrt{U} + \sqrt{U}\,\partial_\mu U\,\partial^\mu U^+\sqrt{U^+})\right.\end{aligned} \tag{3.1}$$

where

$$T = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 0 \end{bmatrix}, \qquad S = \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 1 \end{bmatrix} \tag{3.2}$$

are the projectors onto the nonstrange and strange degrees of freedom, respectively, and $x - 1 \approx 36$ measures the flavor symmetry breaking (Weigel *et al.*, 1990). The parameters β' and δ' are determined from the masses and decay constants of the pion and the kaon and are equal to $\beta' = -26.4$ MeV^2 and $\delta' = 4.15 \times 10^7$ MeV^4. The last term in (3.1) represents the direct derivative coupling of the baryon fields to the photon field A_μ. In fourth-order chiral perturbation, the third term in (3.1) is necessary to reproduce the electromagnetic pion radius correctly in this model, thus determining the value of the parameter $L_9 = (6.9 \pm 0.7) \times 10^{-3}$. It should be noted that the physical pion decay constant (F_π = 186 MeV) is determined by $F_\pi^2 = \overline{F}_\pi^2 - 32\beta'$. In addition to the action obtained using the Lagrangian (3.1), the Wess–Zumino action in the form

$$S = -\frac{iN_c}{240\pi^2} \int d^5x \, e^{\mu\nu\alpha\beta\gamma} \operatorname{Tr}[U^+ \partial_\mu UU^+ \partial_\nu UU^+ \partial_\alpha UU^+ \partial_\beta UU^+ \partial_\gamma U] \tag{3.3}$$

must be included in the total action, where N_c is the number of colors in the underlying QCD. The Wess–Zumino action defines the topological properties of the model, important for the quantization of the solitons. In the $SU(2)$ case the Wess–Zumino action vanishes identically and was therefore not present in the discussions of Sections 1 and 2.

In the $SU(3)$ collective rotational approach the hyperons as chiral solitons are described by the time-dependent meson configuration

$$U = A(t) \sqrt{U_K} \sqrt{U_\pi} \sqrt{U_K} A^+(t) \tag{3.4}$$

where U_π is an $SU(3)$ extension of the usual $SU(2)$ skyrmion field used to describe the nucleon spectrum, and U_K is the field describing the kaons

$$U_\pi = \begin{bmatrix} u_\pi & 0 \\ 0 & 1 \end{bmatrix}, \qquad U_K = \exp[2W(r) d_{i\alpha\beta} r_0^i \Omega_\alpha \lambda_\beta] \tag{3.5}$$

In (3.5) u_π is the usual $SU(2)$ skyrmion field given by (1.4) and $\Omega_\alpha = -i \operatorname{Tr} \lambda_\alpha A^+ \dot{A}$ represents eight angular velocities, where λ_α and d_{abc} are the Gell-Mann matrices and symmetric structure functions of $SU(3)$, respectively. It is also convenient to introduce the adjoint of the collective rotations given by $D_{\alpha\beta} = \frac{1}{2} \operatorname{Tr} \lambda_\alpha A \lambda_\beta A^+$.

The electromagnetic current J_μ^{em} is now obtained in two steps, first by introducing the photon field in the action obtained using (3.1) and (3.3) such that it becomes invariant under the local $U_{\text{em}}(1)$ gauge transformation, and second by identifying J_{em}^μ as an object which couples to photon field linearly. The resulting covariant expression may be found elsewhere (Park *et al.*, 1991; Park and Weigel, 1992). From this expression it is possible to obtain the quadrupole and monopole pieces of the electric and magnetic form factors, respectively. The former is obtained from the orbital angular momentum $l = 2$ term of the time component of the electromagnetic current J_0^{em} and the latter is obtained from the spatial components J_i^{em}. It is therefore suitable to introduce the associated Fourier transforms as follows:

$$\hat{E}(q) = \int_{r>\epsilon} d^3\mathbf{r}\, j_2(qr)\left(\frac{z^2}{r^2} - \frac{1}{3}\right)J_0^{em} \tag{3.6}$$

$$\hat{M}(q) = \frac{1}{2}\int_{r>\epsilon} d^3\mathbf{r}\, j_1(qr)\epsilon^{3ij} r_{0i} J_j^{em} \tag{3.7}$$

Following Abada *et al.* (1996), we obtain in the constant-cutoff approach the results

$$\hat{E}(q) = -\frac{8\pi}{15\alpha^2} D_{em,3}\int_\epsilon^\infty dr\, r^2 j_2(qr)V_0(r) \tag{3.8}$$

$$\begin{aligned}\hat{M}(q) = & -\frac{4\pi}{3}\int_\epsilon^\infty dr\, r^2 j_1(qr)\left\{ V_1(r)D_{em,3}\right.\\ & - \frac{1}{\beta^2} V_2(r) d_{3\alpha\beta} D_{em}^{\alpha} R^{\beta} + V_3(r)D_{88}D_{em,3}\\ & \left. - V_4(r)d^{3\alpha\beta}D_{em,\alpha}D_{8\beta} + \frac{\sqrt{3}}{2\alpha^2} B(r)D_{em,8}R_3\right\}\end{aligned} \tag{3.9}$$

where the moment of inertia into the strange flavor direction β^2 is defined by $R_\alpha = -\beta^2\Omega_\alpha$, and $D_{em,i} = D_{3i} + D_{8i}/\sqrt{3}$.

The function $V_0(r)$ is given by

$$\begin{aligned}V_0(r) = & \frac{1}{4}\sin^2 F\,(F_\pi^2 - 32\beta' \cos F)\\ & - 2L_9\left[\sin 2F\left(\frac{d^2F}{dr^2} + \frac{2}{r}\frac{dF}{dr}\right) + 2\cos 2F\left(\frac{dF}{dr}\right)^2 - 3\,\frac{\sin^2 F}{r^2}\right]\end{aligned} \tag{3.10}$$

while the functions $V_1(r)$, $V_2(r)$, $V_3(r)$, $V_4(r)$, and $B(r)$ are easily obtained from the corresponding functions in (Park *et al.*, 1991; Park and Weigel, 1992), by letting the Skyrme parameter $e \to \infty$, except for the contributions of the third term in (3.1) to $V_1(r)$ and $V_2(r)$, which are obtained in Abada *et al.* (1996).

3.3. Radiative Decay Widths

The radiative decay widths Γ for the decays of the $\frac{3}{2}^+$ baryons to $\frac{1}{2}^+$ baryons are then obtained as matrix elements of $\hat{E}(q)$ and $\hat{M}(q)$, i.e.,

$$\Gamma_{E2}(B \to \gamma B') = \frac{675}{8}\, \alpha_{em} q \,|\langle B'(\tfrac{1}{2}^+)|\hat{E}(q)|B(\tfrac{3}{2}^+)\rangle|^2 \tag{3.11}$$

$$\Gamma_{M1}(B \to \gamma B') = 18\alpha_{em} q \,|\langle B'(\tfrac{1}{2}^+)|\hat{M}(q)|B(\tfrac{3}{2}^+)\rangle|^2 \tag{3.12}$$

where we follow the standard prescription (Abada *et al.*, 1996) and take q to be the momentum of the photon in the rest frame of the $\frac{3}{2}^+$ baryon, and $\alpha_{em} = 1/137$. The matrix elements in (3.11) and (3.12) are calculated in the space of collective coordinates; a detailed account can be found in general in Park *et al.* (1991) and Park and Weigel (1992) and in particular for the decays of the $\Lambda(1405)$ resonance in Schat *et al.* (1995b) and Dalarsson (1996b). Now we are able to calculate the desired E2/M1 ratio as follows:

$$\frac{\text{E2}}{\text{M1}} = \frac{5}{4} \frac{|\langle B'(\frac{1}{2}^+)|\hat{E}(q)|B(\frac{3}{2}^+)\rangle|^2}{|\langle B'(\frac{1}{2}^+)|\hat{M}(q)|B(\frac{3}{2}^+)\rangle|^2} \tag{3.13}$$

Table I compares the numerical predictions of the present model with the results obtained using the CSM (Abada *et al.*, 1996), the NRQM (Darewych *et al.*, 1983), and the QLM (Leinweber *et al.*, 1985) for the same decays as those presented in Abada *et al.* (1996). In the present paper we only consider the complete Lagrangian with the third term in (3.1) included, i.e., for $L_9 = 6.9 \times 10^{-3}$, and use the empirical values for pion and kaon masses and decay constants.

In Table I the results shown in parenthesis, following Abada *et al.* (1996), refer to the case when the ratio E2/M1 is rescaled by the proton magnetic moment. The rescaling of the type E2/M1 $\to$ E2/M1 $\times$ $(\mu_p^{pre}/\mu_p^{exp})$ is motivated by the fact that here, as in the case of the CSM (Abada *et al.*, 1996), the predicted value of the proton magnetic moment is lower than the empirical value. However, in the constant-cutoff approach this rescaling causes a somewhat smaller reduction than in the CSM case (Abada

Table I. Radiative Decay Amplitudes (in keV) and E2/M1 Decay Ratios[a]

	Present results		CSM				Γ_{tot}	
			Set 1		Set 2			
	Γ_{tot}	E2/M1	Γ_{tot}	E2/M1	Γ_{tot}	E2/M1	NRQM	QLM
$\Delta \to \gamma N$	318	−3.7 (−2.9)	313	−3.7 (−2.7)	322	−3.7 (−2.6)	330	430
$\Sigma^{*0} \to \gamma\Lambda$	178	−3.9 (−3.1)	180	−3.8 (−2.8)	194	−3.7 (−2.6)	232	—
$\Sigma^{*-} \to \gamma\Sigma^-$	2	−6.0 (−4.8)	1	−7.3 (−5.3)	2	−4.3 (−3.1)	2	3
$\Sigma^{*0} \to \gamma\Sigma^0$	16	−1.4 (−1.1)	15	−1.5 (−1.1)	12	−1.9 (−1.4)	18	17
$\Sigma^{*+} \to \gamma\Sigma^+$	80	−2.0 (−1.6)	78	−2.2 (−1.6)	71	−2.3 (−1.7)	100	100
$\Xi^{*-} \to \gamma\Xi^-$	4	−4.6 (−3.7)	3	−6.1 (−4.5)	4	−4.3 (−3.0)	3	4
$\Xi^{*0} \to \gamma\Xi^0$	121	−2.0 (−1.6)	115	−2.4 (−1.8)	108	−2.6 (−1.8)	137	129

[a] References for the models: CSM, Abada *et al.* (1996); NRQM, Darewych *et al.* (1983); QLM, Leinweber *et al.* (1985).

et al., 1996). For the calculation of the proton magnetic moment in the constant-cutoff approach see Dalarsson (1993, 1995a–d, 1996a–c) and Schat *et al.* (1995b). It should also be noted that the QLM results given in Leinweber *et al.* (1985) are normalized to fit the magnetic moment of the proton.

From Table I we see that the present results are of the same order of magnitude as the results obtained by other means and there is a general qualitative agreement with the CSM (Abada *et al.*, 1996), NRQM (Darewych *et al.*, 1983), and QLM results (Leinweber *et al.*, 1985).

4. CONCLUSIONS

We have calculated the decay widths for the radiative decays of the $\frac{3}{2}^+$ baryons in the constant-cutoff approach to the collective treatment of the $SU(3)$ Skyrme model by separately evaluating the magnetic dipole (M1) and electric quadrupole (E2) transition matrix elements. As in the CSM (Abada *et al.*, 1996), the total decay widths are strongly dominated by the M1 contribution, giving E2/M1 ratios of the order of few percent only. As in the CSM (Abada *et al.*, 1996), all the ratios are negative.

We have compared the present results with those obtained using other models, the CSM (Abada *et al.*, 1996), NRQM (Darewych *et al.*, 1983), and QLM (Leinweber *et al.*, 1985). Thus we have shown that there is a general qualitative agreement between our results and the results of other models. Regarding the agreement with the only known empirical value for the decay $\Lambda \to \gamma N$, we note that the constant-cutoff approach after rescaling is somewhat more in variance with the empirical value than the CSM (Abada *et al.*, 1996).

On the other hand, the constant-cutoff approach employed in this paper offers a simpler analytical structure of the results and less complicated calculations of the quantities which describe the strong and electromagnetic properties of hyperons (Dalarsson, 1993, 1995a–d, 1996a–c).

Finally, it should be noted that the empirical values for most of the calculated quantities are unfortunately difficult to obtain. As argued in Abada *et al.* (1996), better empirical information about the radiative decay processes is needed in order to determine the quality of predictions of different models. Some experiments to that effect are being prepared at several experimental facilities (see Abada *et al.*, 1996, and references therein).

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